Sri Lanka Institute of Information Technology

IT0060 –Essential Mathematics

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Simultaneous Equations and Quadratic Equations

Introduction

- Sometimes you'll encounter problems with:
  - Two or more **unknown quantities**
  - Two or more **equations** relating them

- **What are Simultaneous Equations?**
  - Equations that must be satisfied **at the same time**
  - You need to find values of the unknowns that work in **ALL** given equations simultaneously

Understanding the Solution

- Consider these simultaneous equations:

$$3x + 2y = 36$$
$$5x + 4y = 64$$

- **Solution**

$$\mathbf{x = 8 \text{ and } y = 6}$$

- **Verification:**

Equation 1: $3(8) + 2(6) = 24 + 12 = 36$ ✓

Equation 2: $5(8) + 4(6) = 40 + 24 = 64$ ✓

The Elimination Method

- A process of **removing (eliminating)** one unknown to leave a single equation with the other unknown

- **1st Step - Multiply** equations to match coefficients
- **2nd Step - Add or subtract** to eliminate one variable
- **3rd Step - Solve** for the remaining variable
- **4th Step - Substitute** back to find the other variable

The Elimination Method

- **Example 1: Step-by-Step**
- Solve the simultaneous equations:

$$3x + 2y = 36 \text{ .... } (1)$$
$$5x + 4y = 64 \text{ .... } (2)$$

✓**Strategy:** Eliminate y by making coefficients equal

Notice: equation (2) has 4y, which is 2 × 2y from equation (1)

✓**Multiply** equation (1) by 2:

$$6x + 4y = 72 \text{ .... } (3)$$

The Elimination Method

- **Example 1: Elimination**

✓**Subtract** equation (2) from equation (3):

$$\begin{array}{r} 6x + 4y = 72 \;...\; (3) \\ -\;(5x + 4y = 64) \;...\; (2) \\ \hline x + 0y = 8 \end{array}$$

x = 8

- *We've eliminated y and found x!*
- Substitute **x=8** to equation (1),

$3(8) + 2y = 36$

$24 + 2y = 36$

y = 6

The Elimination Method

- **Example 2: Elimination by Addition**
  - Solve the simultaneous equations:

$$5x - 3y = 26 \quad \ldots (1)$$
$$4x + 2y = 34 \quad \ldots (2)$$

✓ **Strategy:** Eliminate y

  ✓ We need to make the y coefficients equal (but opposite signs)

✓ **Multiply:**

  - Equation (1) × 2: $10x - 6y = 52$ ... (3)
  - Equation (2) × 3: $12x + 6y = 102$ ... (4)

The Elimination Method

- **Example 2: Adding Equations**

✓**Add** equations (3) and (4):

$$10x - 6y = 52 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$
$$+\ (12x + 6y = 102) \quad \ldots\ldots\ldots\ldots\ldots (4)$$

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$$22x + 0y = 154$$
$$22x = 154$$
$$x = 154 \div 22 = \mathbf{7}$$

- *Notice: Adding eliminated y because −6y + 6y = 0*
- Substitute **x=7** to equation (2),

$$4x+3y=34$$
$$4(7) + 3y = 34$$
$$\mathbf{y=2}$$

Practice Exercises

- Try solving these simultaneous equations:

a) $7x + y = 25$
$5x - y = 11$

b) $8x + 9y = 3$
$x + y = 0$

c) $2x + 13y = 36$
$13x + 2y = 69$

d) $7x - y = 15$
$3x - 2y = 19$

What is a Quadratic Equation?

General Form

$$ax^2 + bx + c = 0$$

Key Characteristics:

- Must contain an x^2 term (e.g., $3x^2$, $-5x^2$, or x^2)
- May contain an x term (e.g., 5x, -7x, 0.5x)
- May contain a constant term (e.g., 6, -7, ½)
- Cannot have x^3 or higher powers
- Cannot have terms like 1/x
- The coefficient 'a' cannot be zero

Three Methods to Solve Quadratic Equations

1. Factorisation

Break down the equation into factors and solve

2. Completing the Square

Rewrite as a perfect square to isolate x

3. Quadratic Formula

Use the universal formula for any quadratic

Method 1: Factorization

Example 1: $3x^2 = 27$

Step 1: Write in standard form

$3x^2 - 27 = 0$

Step 2: Factor out common factor

$3(x^2 - 9) = 0$

Step 3: Recognize difference of squares

$3(x - 3)(x + 3) = 0$

Step 4: Solve for x

$x = 3$ or $x = -3$

Example 2: $x^2 - 5x + 6 = 0$

Find two numbers that:

- Multiply to give 6
- Add to give -5

Numbers: -3 and -2

Factorised form:

$(x - 3)(x - 2) = 0$

Solutions:

$x = 3$ or $x = 2$

Method 2: Completing the Square

Example: $x^2 - 3x - 2 = 0$

Step 1: Take half the coefficient of x and square it
Half of -3 is -3/2, squared gives 9/4

Step 2: Rewrite as a perfect square
$(x - 3/2)^2 - 9/4 - 2 = 0$

Step 3: Simplify
$(x - 3/2)^2 - 17/4 = 0$
$(x - 3/2)^2 = 17/4$

Step 4: Take square root and solve
$x - 3/2 = \pm\sqrt{17}/2$

Solutions:
$x = (3 + \sqrt{17})/2$ or $x = (3 - \sqrt{17})/2$

Method 3: The Quadratic Formula

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the equation: $ax^2 + bx + c = 0$

Example: $3x^2 - 5x + 1 = 0$

Identify: a = 3, b = -5, c = 1

Substitute into formula:

$$x = \frac{-5 \pm \sqrt{5^2 - 12}}{2 \times 3} = \frac{-5 \pm \sqrt{25 - 12}}{6} = \frac{-5 \pm \sqrt{13}}{6} = \frac{-5 \pm 3.6056}{6}$$

$$x = \frac{-5 + 3.6056}{6} = -0.2324 \quad OR \quad x = \frac{-5 - 3.6056}{6} = -1.4343$$

This formula works for ANY quadratic equation!

The Discriminant: $b^2 - 4ac$

The discriminant ($b^2 - 4ac$) in the quadratic formula determines the nature of the roots:

$b^2 - 4ac > 0$
Two distinct real roots (graph crosses x-axis twice)

$b^2 - 4ac = 0$
One repeated real root (graph touches x-axis once)

$b^2 - 4ac < 0$
No real roots (graph doesn't touch x-axis)

Exercises

1. Use factorization to solve the following quadratic equations.
   a) $x^2 + 19x + 60 = 0$
   b) $2x^2 + x - 6 = 0$
   c) $2x^2 - x - 6 = 0$
   d) $4x^2 = 11x - 6$
2. Use completing the square to solve:
   a) Show that $x^2 + 2x = (x + 1)^2 - 1$. Hence, use completing the square to solve $x^2 + 2x - 3 = 0$.
   b) Show that $x^2 - 6x = (x - 3)^2 - 9$.Hence use completing the square to solve $x^2 - 6x = 5$.
   c) $x^2 - 5x + 1 = 0$.
   d) $x^2 + 8x + 4 = 0$.
3. Use the quadratic formula to solve the following quadratic equations.
   a) $x^2 - 3x + 2 = 0$
   b) $4x^2 - 11x + 6 = 0$
   c) $x^2 - 5x - 2 = 0$
   d) $3x^2 + 12x + 2 = 0$
   e) $2x^2 = 3x + 1$
   f) $x^2 + 3 = 2x$
   g) $x^2 + 4x = 10$
   h) $25x^2 = 40x - 16$